

# View in 2D & 3D

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#### Outline

- 2D Viewing
- 3D Viewing
  - Classic view
  - Computer view
    - Positioning the camera
    - Projection



- The world is infinite (2D or 3D) but the screen is finite
- Depending on the details the user wishes to see, he limits his view by specifying a window in this world





## 2D Viewing

 By applying appropriate transformations we can map the world seen through the window on to the screen



2D World





## Windowing Concepts

- Window is a rectangular region in the 2D world specified by
  - a *center (xCenter, yCenter)* and
  - size windowSize
- Screen referred to as Viewport is a discrete matrix of pixels specified by
  - size screenSize (in pixels)





## 2D Viewing Transformation

- Mapping the 2D world seen in the window on to the viewport is 2D viewing transformation
  - also called *window to viewport transformation*





## **Deriving Viewport Transformation**





## **Deriving Viewport Transformation**



 Given any point in the 2D world, the above transformations maps that point on to the screen



# The Aspect Ratio (纵横比)

- In 2D viewing transformation the *aspect ratio* is maintained when the scaling is uniform
- scaleFactor is same for both x and y directions





## Maintaining the Aspect





## gluOrtho2D( left, right, bottom, top )

Creates a matrix for projecting 2D coordinates onto the screen and multiplies the current matrix by it.

## glViewport( x, y, width, height )

Define a pixel rectangle into which the final image is mapped. (x, y) specifies the lower-left corner of the viewport. (width, height) specifies the size of the viewport rectangle.



## **OpenGL** Commands





**Computer Graphics** 

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## **3D Viewing**

- To display a 3D world onto a 2D screen
  - Specification becomes complicated because there are many parameters to control

 Additional task of reducing dimensions from 3D to 2D (projection)

 3D viewing is analogous to taking a picture with a camera





The principle upon which all camera equipment works is traced to artist / inventor Leonardo da Vinci who showed that all that was needed to project an image was a small pinhole through which light could pass. The smaller the hole the sharper the image.

The basic camera, called a pinhole camera, existed in the early 17th Century. It took much longer for science to find a light sensitive material to record the image. It was not until 1826 when Joseph Niepce from France discovered that silver chloride (氯化银) could be used to make bitumen sensitive to light.



The world through a pinhole

http://www.phy.ntnu.edu.tw/java/pinHole/pinhole.html



#### Modeling transformation

• Shaping, positioning and moving the objects in the world scene

#### Viewing transformation

• Positioning and pointing camera onto the scene, selecting the region of interest

#### Projection transformation

• Adjusting the distance of the eye

#### Viewport transformation

• Enlarging or reducing the physical photograph



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#### **Classic View**

- Three basic elements needed
  - One or more objects
  - Observer with a projection plane
  - Projection transform: from the object to the projection plane
- The classic view is based on the relationship between these elements



#### **Plane Geometry Projection**

- That is projected onto the **plane** of the standard projection
- Projection line is a straight line
  - Gathering in the center of projection
  - Parallel to each other
- This projection preserve collinearity
- Some application such as mapping (地图映射)need to be non planar projection



### **Taxonomy of Projection**



Figures extracted from Angle's textbook

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Computer Graphics 2014, ZIU



## Orthogonal projection (正交投影)

• Projection line perpendicular (垂直) to the plane of projection





## Multi-view orthographic projection (多角度正交投影)

- The projection plane parallel to the reference plane(DOP is perpendicular to the view plane)
- Usually projection from the front, top and side





#### Multi-view of CAD parts

在CAD和建筑行业中,通常 显示出来三个视点图以及等 角投影图。







#### Advantages & Disadvantages

- Keep the distance and angle
  - Remain the shapes
  - Use for measurement (building, manual)
- Can't see the global real object shape, because many surface not visible in view
  - Sometimes adding isometric drawing (等角图)





### Axonometric projection (轴测投影)

• DOP orthogonal to the projection plane, but...

... orient projection plane with respect to the object

 Parallel lines remain parallel, and receding lines are equally foreshortened by some factor.



Projection type depends on angles made by projector with the three principal axes.

Figures extracted from Angle's textbook



### **Mechanical Drawing**



isometric 等角图

A ≠ B ≠ C
trimetric 不等角图

### Oblique Projections (斜平行投影)

- Most general parallel views
- Projectors make an arbitrary angle with the projection plane
- Angles in planes parallel to the projection plane are preserved



cavalier

斜等测

#### Cavalier

Angle between projectors and projection plane is 45°. Perpendicular faces are projected at full scale

#### Cabinet

Angle between projectors and projection plane is 63.4°. Perpendicular faces are projected at 50% scale

斜二测







#### Perspective projection





#### Giotto di Bondone (1267~1337)

这位13世纪末、14世纪初的画 家,为公认的西方绘画之父、文 艺复兴的先驱。但以往因意大利 官方难得愿意出借国宝外展,因 此国人多不够熟悉这位西方美术 史上的经典人物。



Not systematic -- lines do not converge to a single "vanishing" point

Computer Graphics 2014, ZIU



### Vanishing Points

- On the object of all parallel lines (not parallel to the projection plane) projected to a point
- Hand draw simple perspective projection on the need to use the vanishing point





### Vanishing Points

#### Brunelleschi



O

#### Brunelleschi's Peepshow (西洋镜)

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Filippo Brunelleschi



0Z%

### **Perspective Projection**

- Characterized by diminution of size
- The farther the object, the smaller the image
- Foreshortening depends on distance from viewer
- Can't be used for measurements
- Vanishing points









#### **Perspective Viewing**









2. 两点透视 画面与建筑物的高度方向的棱线平行,而与另外两组棱线的方向倾斜,此时,长度方向和宽度方向的棱线各有一个灭点。






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The fundamental difference between the classic view and computer view:

- All the classical views are based on a particular relationship among the objects (对象), the viewers (观察者), and the projectors (投影线).
- In computer graphics, we stress the independence of the object specifications (对象 定义) and camera parameters (摄像机参数).



## Computer view

- The view has three functions, are implemented in pipeline system
  - Positioning the camera
    - Setup the model-view matrix
  - Set the lens
    - Projection matrix
  - Clipping
    - view frustum





## Camera in OpenGL

- In OpenGL, the initial world frame and camera frame are the same
- A camera located at the origin, and point to the negative direction of Z axis
- OpenGL also specifies the view frustum default, it is a center at the origin of the side length of 2 cube





#### Default projection

• Default is the projection of orthogonal projection





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### Moving the camera frame

- If you want to see objects with positive Z coordinate more, we can
  - Moves The camera along the positive Z axis
  - Moves the object along the negative Z axis
- They are equivalent, is determined by the model-view matrix
  - Need a translation: glTranslated(0.0, 0.0, d);
  - Here, d > 0



#### Moving the camera frame





### Moving the camera frame

- Can use a series of translation and rotation to the camera position to any position
- For example, in order to get the side view
  - Rotate the camera: R
  - Move the camera from the origin: T
  - C = TR





# **Viewing Specification**



All the specifications are in world coordinates



## View Reference Coordinate System





### View Reference Coordinate System



 Once the view reference coordinate system is defined, the next step is to project the 3D world on to the view reference plane



## Simplest Camera Position

- Projecting on to an arbitrary view plane looks tedious
- One of the simplest camera positions is one where vrp coincides y with the world origin and

```
u,v,n matches x,y,z
```

 Projection could be as simple as ignoring the z-coordinate





### World to Viewing coordinate Transformation

- The world could be transformed so that the view reference coordinate system coincides with the world coordinate system
- Such a transformation is called world to viewing coordinate transformation
- The transformation matrix is also called view orientation matrix



#### **Deriving View Orientation Matrix**



• The *view orientation matrix transforms* a point from *world coordinates* to *view coordinates* 

$$\begin{bmatrix} u_x & u_y & u_z & -\overset{1}{u} \bullet vrp \\ v_x & v_y & v_z & -\overset{r}{v} \bullet vrp \\ n_x & n_y & n_z & -\overset{r}{n} \bullet vrp \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



### A More Intuitive Approach Offered by GLU

OpenGL provides a very helpful utility function that implements the look-at viewing specification:

gluLookAt	(	eyex,	eyey,	eyez,	//	eye point
		at <b>x</b> ,	aty,	atz,	//	lookat point
		upx,	upy,	upz );	//	up vector

These parameters are expressed in world coordinates



gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)

postmultiplies current matrix, so to be safe:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
// now ok to do model transformations
```

它封装了世界坐标系到观察坐标系的转换。调用之后,我们就把坐标 系变换的矩阵放入了矩阵栈,后续对物体的位置描述,会通过此矩阵 栈进行转换到我们的观察坐标系了。



# gluLookAt Illustration





## Look-At Positioning

- We specify the view frame using the look-at vector a and the camera up vector up
- The vector a points in the negative viewing direction



In 3D, we need a third vector that is perpendicular to both up and a to specify the view frame



## Where does it point to?

- The result of the cross product is a vector, not a scalar, as for the dot product
- In OpenGL, the cross product **a** × **b** yields a RHS vector.
   **a** and **b** are the thumb and index fingers, respectively





## **Constructing a Coordinates**

The cross product between the up and the look-at vector will get a vector that points to the right.



- r = up × a
- Finally, using the vector **a** and the vector **r** we can synthesize a new vector **u** in the up direction:





### Rotation

 Rotation takes the unit world frame to our desired view reference frame:

$$\begin{bmatrix} \mathbf{r}_{\mathbf{x}} & \mathbf{u}_{\mathbf{x}} & \mathbf{a}_{\mathbf{x}} & \mathbf{0} \\ \mathbf{r}_{\mathbf{y}} & \mathbf{u}_{\mathbf{y}} & \mathbf{a}_{\mathbf{y}} & \mathbf{0} \\ \mathbf{r}_{\mathbf{z}} & \mathbf{u}_{\mathbf{z}} & \mathbf{a}_{\mathbf{z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{R}$$





### Translation

Translation to the eye point:





## Composing the Result

• The final viewing coordinate transformation is:

$$\mathbf{E} = \mathbf{T}\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & eye_x \\ 0 & 1 & 0 & eye_y \\ 0 & 0 & 1 & eye_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x & u_x & a_x & 0 \\ r_y & u_y & a_y & 0 \\ r_z & u_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# The Viewing Transformation

■ Transforming all points P in the world with E<sup>-1</sup>:

$$\mathbf{V} = \mathbf{R}^{-1}\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{r}_{x} & \mathbf{r}_{y} & \mathbf{r}_{z} & \mathbf{0} \\ \mathbf{u}_{x} & \mathbf{u}_{y} & \mathbf{u}_{z} & \mathbf{0} \\ \mathbf{a}_{x} & \mathbf{a}_{y} & \mathbf{a}_{z} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & -\operatorname{eye}_{x} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -\operatorname{eye}_{y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & -\operatorname{eye}_{z} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Where these are normalized vectors:

$$\mathbf{a} = P_{eye} - P_{look}$$
  
 $\mathbf{r} = \mathbf{u}\mathbf{p} \times \mathbf{a}$   
 $\mathbf{u} = \mathbf{a} \times \mathbf{r}$ 



## Looking At a cube

Setting up the OpenGL look-at viewing transformation:



### gluLookAt() and other transformations

- The user can define the model-view matrix to achieve the same function
- But from the concept of the gluLookAt () as the camera position, while the other follow-up transformation as object position
- gluLookAt in the OpenGL () function is the only specialized for positioning the camera function



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- I. Apply the world to view transformation
- Apply the parallel projection matrix to project the 3D world onto the view plane

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} u_x & u_y & u_z & -\stackrel{\mathbf{r}}{u} \bullet vrp \\ v_x & v_y & v_z & -\stackrel{\mathbf{r}}{v} \bullet vrp \\ n_x & n_y & n_z & -\stackrel{\mathbf{r}}{n} \bullet vrp \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Apply 2D viewing transformations to map the view window on to the screen



#### Orthogonal Projection Matrix: Homogeneous coordinates

$$\begin{split} \mathbf{P}_{p} &= \mathbf{M}\mathbf{p} & x_{p} = x \\ \mathbf{M} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & y_{p} = y \\ z_{p} &= 0 \\ w_{p} &= 1 \\ & & & \\ \hline \mathbf{E}_{g} \mbox{$\mathbb{K}$} \mbox{$\mathbb{K}$$$



## **Perspective Projection**

- The points are transformed to the view plane along lines that converge to a point called
  - projection reference point (prp) or
  - center of projection (cop)
- *prp* is specified in terms of the viewing coordinate system



**Transformation Matrix for Perspective Projection** 

*prp* is usually specified as perpendicular distance d behind the view plane



transformation matrix for perspective projection

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$



#### **Perspective Projection**





#### **Basic Perspective Projection**



Given p = Mq, write out the Projection Matrix M.



### Homogeneous Coordinates

 $\mathbf{p} = \mathbf{M}\mathbf{q}$ 

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \Rightarrow \qquad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$



## **Perspective Divide**

- 如果w ≠ 1, 那么必须从齐次坐标中除以w而得到所表示的 点
- 这就是透视除法,结果为

$$x_p = \frac{x}{z/d}, \quad y_p = \frac{y}{z/d}, \quad z_p = d$$



## **Perspective Projection**

- Perspective Divide 是非线性的,导致非均匀缩短。
  - 离投影中心(COP)远的对象投影后,尺寸缩短得比离COP近的对象大。
- •透视变换是**保直线**的,但不是**仿射变换**。
- 透视变换是不可逆的,因为沿一条投影直线上的所有点投影后的结果相同。


# **Perspective Projection**

I.Apply the view orientation transformation

- 2. Apply translation, such that the center of the view window coincide with the origin
- 3. Apply the perspective projection matrix to project the 3D world onto the view plane

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -cx \\ 0 & 1 & 0 & -cy \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} u_x & u_y & u_z & -\overset{\mathbf{r}}{u} \bullet vrp \\ v_x & v_y & v_z & -\overset{\mathbf{r}}{v} \bullet vrp \\ n_x & n_y & n_z & -\overset{\mathbf{r}}{n} \bullet vrp \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Apply 2D viewing transformations to map the view window (centered at the origin) on to the screen



### **View Window**

- **View window** is a rectangle in the view plane specified in terms of view coordinates.
- Specify center (cx, cy), width and height
- prp lies on the axis passing through the center of the view window and parallel to the n-axis





# View Volume & Clipping

- For perspective projection the view volume is a semi-infinite pyramid with apex (顶点) at prp and edges passing through the corners of the view window
- For efficiency, view volume is made finite by specifying the front and back clipping plane specified as distance from the view plane





# View Volume & Clipping

• For parallel projection the *view volume* is an infinite parallelepiped (平行六面体) with sides parallel to the direction of projection

 View volume is made finite by specifying the front and back clipping plane specified as distance from the view plane



 Clipping is done in 3D by clipping the world against the front clip plane, back clip plane and the four side planes



#### The Complete View Specification

#### Specification in world coordinates

- position of viewing (*vrp*), direction of viewing(-*n*),
- up direction for viewing (*upVector*)
- Specification in view coordinates
  - view window : center (*cx, cy*),
    *width* and *height*,
  - *prp* : distance from the view plane,
  - front clipping plane : distance from view plane
  - back clipping plane : distance from view plane

**Computer Graphics** 





### glOrtho(left, right, bottom, top, near, far);





#### Perspective in OpenGL

### glFrustum(left, right, bottom, top, near, far);



glMatrixMode(GL\_PROJECTION);

glLoadldentity( );

glFrustum(left, right, bottom, top, near, far);



# gluPerspective(fovy, aspect, near, far);



FOV is the angle between the top and bottom planes



# Field of application

- Application of glFrustum sometimes difficult to get the desired results
- GluPerspective (fovy, aspect, near, far) can provide a better results





# Normalization

- Normalization allows for a single pipeline for both perspective and orthogonal viewing.
- It simplifies clipping.
- Projection to the image plane is simple (discard z).
- z is retained for z-buffering (visible surface determination)



#### glOrtho(left, right, bottom, top, near, far)



## **Transformation Pipeline**





1. Vertices of the Object to draw are in **Object space** (as modelled in your 3D Modeller)

2. ... get transformed into World space by multiplying it with the Model Matrix

3. Vertices are now in **World space** (used to position the all the objects in your scene)

4. ... get transformed into Camera space by multiplying it with the **View Matrix** 

5. Vertices are now in View Space

– think of it as if you were looking at the scene throught "the camera"

6. ... get transformed into Screen space by multiplying it with the **Projection Matrix** 

7. Vertex is now in **Screen Space** – This is actually what you see on your Display.



# **Projective Rendering Pipeline**



